Further Maths Revision Paper 3

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q1 and 3)

1

Use the t-formula to solve

$$3\sin\theta - 2\cos\theta = 1$$

in the interval $0° \leqslant \theta \leqslant 360°$

$\sin\theta = \frac{2t}{1+t^2}$ $\quad \cos\theta = \frac{1-t^2}{1+t^2}$ $\quad 0° \le \frac{1}{2}\theta \le 180°$

$$\frac{6t}{1+t^2} - \frac{2-2t^2}{1+t^2} = 1$$

$$6t - 2 + 2t^2 = 1 + t^2$$

$$t^2 + 6t - 3 = 0$$

$$t = -3 \pm 2\sqrt{3}$$

$\tan\frac{1}{2}\theta = t$

$\tan\frac{1}{2}\theta = 2\sqrt{3} - 3$

$\frac{1}{2}\theta = 24.9°,$

$\underline{\underline{\theta = 49.8°}}$

$\tan\frac{1}{2}\theta = -2\sqrt{3} - 3$

$\frac{1}{2}\theta = 98.8°$

$\underline{\underline{\theta = 197.6°}}$

2

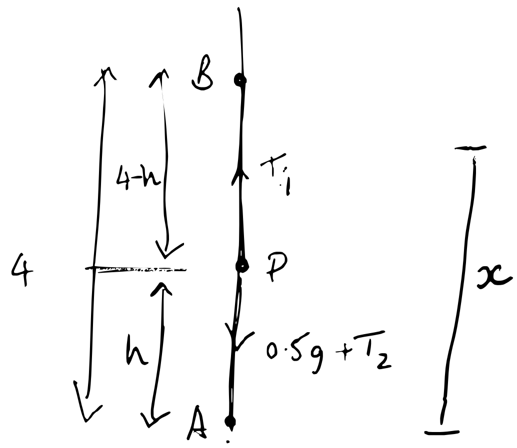
Two identical elastic strings of length 1m and modulus of elasticity 4.9N are each attached to a particle of mass 0.5kg.
Their other two ends are fixed to two points 4m apart in a vertical line.

(a) Find the height of the particle above the lower fixed point A in the equilibrium position.

The particle is now pulled down to A and released from rest.

(b) Find the greatest height above A to which the particle rises

a)



$$0.5g + T_2 = T_1$$

$$0.5g + \frac{\lambda(h-1)}{1} = \frac{\lambda(3-h)}{1}$$

$$0.5g = 3\lambda - \lambda h - \lambda h + \lambda$$

$$0.5g - 4\lambda = -2\lambda h$$

$$\frac{0.5g - 4\lambda}{-2\lambda} = h \qquad h = 1.5m$$

b)

$$EPE = GPE + EPE$$

$$\frac{4.9(3)^2}{2} = (0.5)(9.8)x + \frac{4.9(x-1)^2}{2}$$

$$\frac{9}{2} = 0.5(2)x + \frac{(x-1)^2}{2}$$

$$9 = 2x + (x-1)^2$$

$$9 = 2x + x^2 - 2x + 1$$

$$x^2 = 8$$

$$x = 2.83m$$

3

(a) Verify that the complex number $\alpha = e^{\frac{2\pi i}{5}}$ is a root of the equation $z^5 - 1 = 0$.

(b) Show that $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

(c) Find a quadratic equation whose roots are $\alpha + \alpha^4$ and $\alpha^2 + \alpha^3$

(d) Hence, or otherwise , show that

$$\cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$$

a) $\left(e^{\frac{2\pi i}{5}}\right)^5 - 1$

$= e^{2\pi i} - 1$

$= 1 - 1$

$= 0$.

$\frac{a(1-r^n)}{1-r}$

b) $1 + e^{\frac{2\pi i}{5}} + e^{\frac{4\pi i}{5}} + e^{\frac{6\pi i}{5}} + e^{\frac{8\pi i}{5}}$

$= \frac{1\left(1-\left(e^{2\pi i/5}\right)^5\right)}{1 - e^{2\pi i/5}} = 0$.

c) Sum of roots $= -1$

Product of roots $(\alpha+\alpha^4)(\alpha^2+\alpha^3)$

$= \alpha^3 + \alpha^6 + \alpha^4 + \alpha^7$

$= \alpha^3 + \alpha^4 + \alpha^6 + \alpha^7$

$= \alpha^3(1+\alpha+\alpha^3+\alpha^4)$

$= \alpha^3(-\alpha^2)$

$= -\alpha^5$

$= -1$

$z^2 + z - 1 = 0$

d) $z^2 + z - 1 = 0$

$z = \frac{-1 \pm \sqrt{1-4(1)(-1)}}{2}$

$z = \frac{-1 \pm \sqrt{5}}{2}$

$\cos\frac{2\pi}{5} + \cos\frac{8\pi}{5} = \frac{-1+\sqrt{5}}{2}$

$\cos\frac{2\pi}{5} + \cos\left(-\frac{2\pi}{5}\right) = \frac{-1+\sqrt{5}}{2}$

$2\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4}$

(a) The roots of the equation $x^3 + px^2 + qx - 30 = 0$ are in the ratio $2 : 3 : 5$
Find p and q.

(b) If the roots of the equation

$$4x^3 + 7x^2 - 5x - 1 = 0$$

are α, β, γ find the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$

a) $\alpha : \beta : \gamma = 2:3:5$ $\qquad \alpha\beta\gamma = 30$

$\alpha + \beta + \gamma = 10$

$\alpha\beta + \alpha\gamma + \beta\gamma = 6 + 15 + 10$

$= 31$

$x^3 - 10x^2 + 31x - 30$ $\qquad p = -10 \quad q = 31$

b) $\sum\alpha = -\frac{7}{4}$ $\qquad \sum\alpha\beta = \frac{5}{4}$ $\qquad \sum\alpha\beta\gamma = +\frac{1}{4}$

$u = \alpha\beta, \beta\gamma, \gamma\alpha$

$x = \gamma, \alpha, \beta$

$u = \frac{\alpha\beta\gamma}{x} = +\frac{1}{4x}$ $\qquad x = +\frac{1}{4u}$

$4\left(\frac{+1}{4u}\right)^3 + 7\left(\frac{+1}{4u}\right)^2 - 5\left(\frac{+1}{4u}\right) - 1 = 0$

$+\frac{1}{16u^3} + \frac{7}{16u^2} - \frac{5}{4u} - 1 = 0$

$+1 + 7u - 20u^2 - 16u^3 = 0$

5

Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 1$ which make equal angles with the axes.

$$3x^2 - 4y^2 = 1 \qquad \frac{x^2}{1/3} - \frac{y^2}{1/4} = 1$$

$$x = \frac{1}{\sqrt{3}}\sec\theta \quad y = \frac{1}{2}\tan\theta$$

$$\frac{dx}{d\theta} = \frac{1}{\sqrt{3}}\sec\theta\tan\theta \qquad \frac{dy}{d\theta} = \frac{1}{2}\sec^2\theta$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}\sec^2\theta}{\frac{1}{\sqrt{3}}\sec\theta\tan\theta}$$

$$= \frac{\sqrt{3}}{2}\frac{\sec\theta}{\tan\theta}$$

$$= \frac{\sqrt{3}}{2}\cot\theta\sec\theta$$

$$= \frac{\sqrt{3}}{2}\operatorname{cosec}\theta$$

$$\frac{\sqrt{3}}{2}\operatorname{cosec}\theta = 1 \qquad \frac{\sqrt{3}}{2}\operatorname{cosec}\theta = -1$$

$$\operatorname{cosec}\theta = \frac{2}{\sqrt{3}} \qquad \theta = -\frac{\pi}{3}, -\frac{2\pi}{3}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = \frac{1}{\sqrt{3}}\frac{1}{\cos\frac{\pi}{3}} \quad y = \frac{1}{2}\tan\frac{\pi}{3} \qquad x = \frac{+2}{\sqrt{3}},\ y = -\frac{\sqrt{3}}{2}$$

$$x = \frac{2}{\sqrt{3}} \quad y = \frac{\sqrt{3}}{2} \qquad x = -\frac{2}{\sqrt{3}},\ y = \frac{-\sqrt{3}}{2}$$

$$x = \frac{-2}{\sqrt{3}} \quad y = \frac{\sqrt{3}}{2}$$

$$y - \frac{\sqrt{3}}{2} = x - \frac{2}{\sqrt{3}}$$

$$\boxed{\begin{aligned} y &= x - \frac{\sqrt{3}}{6} \\ y &= x + \frac{\sqrt{3}}{6} \\ y &= -x - \frac{\sqrt{3}}{6} \\ y &= -x + \frac{\sqrt{3}}{6} \end{aligned}}$$